Solution of Fuzzy Game Problem by using Dodecagonal Fuzzy Numbers

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Abstract

The fuzzy set theory has been applied in almost every business enterprise as well as day to day activity. Ranking of fuzzy numbers plays an important role in decision making process. In this paper, we introduced a fuzzy game problem (FGP) in which the values of payoff matrix are represented by dodecagonal fuzzy numbers. By using ranking to payoffs we convert the fuzzy game problem into crisp valued game problem, which can be solved by traditional method.

Keywords: Fuzzy numbers, Dodecagonal Fuzzy Numbers, Fuzzy Game Problem, Fuzzy Ranking

1. INTRODUCTION:

The concept of fuzzy set theory deals with imprecision, vagueness in real life situations. It was firstly proposed by Zadeh [1]. Bellman and Zadeh [2] elaborated on the concept of decision making in the fuzzy environment. Later on, fuzzy methodologies have been successfully applied in a wide range of real world situations. Jain [3] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager [4] used the concept of centroids in the ranking of fuzzy numbers.

Game theory is a mathematical tool that deals with decision making in situations of conflict and cooperation between intelligent and rational decision-makers. In a game problem each player attempts to take best decision by selecting various strategies from the set of available strategies.

Theory of games has played an important role in decision making fields such as defence, economics, political science, management etc. Game theory owes its origin to Mathematician John von Neumann and Economist Oskar Morgenstren .

The traditional game theory assumes the existence of exact payoffs to solve competitive situations. However in the real life game situations such precise information on the payoffs is not available. Due to lack of information, the players are not able to estimate exactly payoffs in real situations. This lack of certainty may be appropriately modelled by using fuzzy set.

In Fuzzy Game Problems, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular, trapezoidal, hexagon, octagon etc. Hussian and Priya [5] discussed the solution of fuzzy game problem with pure strategies by using maximin – minimax principle. Jatinder and Neha [6] proposed the ranking method for ordering dodecagonal fuzzy numbers based on rank, mode, divergence and spread. Jatinder and Neha [7] described an approach fir solving fuzzy transportation problem using dodecagonal fuzzy numbers. Selvakumari and lavanya [8] considered an approach for solving fuzzy game problem in which payoff are triangular and trapezodial fuzzy numbers. Selvakumari and lavanya [9] considered a ranking of octagonal fuzzy numbers to solve fuzzy game problem. After applying ranking, these fuzzy numbers are converted into crisp problem and then solved by traditional method.

2. PRELIMINARIES

In this section, definitions of fuzzy set, fuzzy numbers and dodecagonal fuzzy numbers are presented.

Definition 2.1 Let X = {x} denote a collection of objects denoted generically by x. Then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A}$ = {(x, $\mu_{\widetilde{A}}$ (x)); x ∈ X} where $\mu_{\widetilde{A}}$(x) is termed as the grade of membership of x in A and $\mu_{\widetilde{A}}$: X → M is a function from X to a space M which is called membership space. When M contains only two points, 0 and 1, A is non fuzzy and its membership function becomes identical with the characteristic function of a non fuzzy set.

Definition 2.2 A Fuzzy set $\widetilde{A}$ of universe set X is normal if and only if $Sup_{x \epsilon X}\ \mu_{\widetilde{A}}$ (x) =

Definition 2.3 A fuzzy set $\widetilde{A}$ in universal set X is called convex iff

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)] \text{ for all } x_1, x_2 \in X \text{ and } \lambda \in [0,1]$$

Definition 2.4 A fuzzy set $\widetilde{A}$ of universal set is a fuzzy number iff it is normal and convex.

Definition 2.5 A fuzzy number

$\widetilde{A}$ = (m, n, α, β)$_{LR}$ is said to be an LR flat fuzzy number if its membership function is given by:

$$\mu_{\widetilde{A}}(x) = \begin{Bmatrix} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & x \geq n, \beta > 0 \\ 1, & m \leq x \leq n \end{Bmatrix}$$

L and R are called reference functions, which are continuous, non-increasing functions that defining the left and right shapes of $\mu_{\widetilde{A}}$ (x) respectively and

L(0)=R (0) =1.

3. Dodecagonal Fuzzy Numbers:

A generalised fuzzy number $\widetilde{A_D}$ =($a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}; w$) is said to be dodecagonal fuzzy number if its membership function $\mu_{\widetilde{A_D}}$(x) is given below:

$$\mu_{\widetilde{A_D}}(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (1-k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ k_2 + (1-k_2)\left(\frac{a_8-x}{a_8-a_7}\right) & a_5 \le x \le a_6 \\ 1 & a_6 \le x \le a_7 \\ k_2 + (1-k_2)\left(\frac{x-a_5}{a_6-a_5}\right) & a_7 \le x \le a_8 \\ k_2 & a_8 \le x \le a_9 \\ k_1 + (1-k_1)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ k_1 & a_{10} \le x \le a_{11} \\ k_1\left(\frac{a_2-x}{a_2-a_1}\right) & a_{11} \le x \le a_{12} \\ 0 & x \ge a_{12} \end{cases}$$

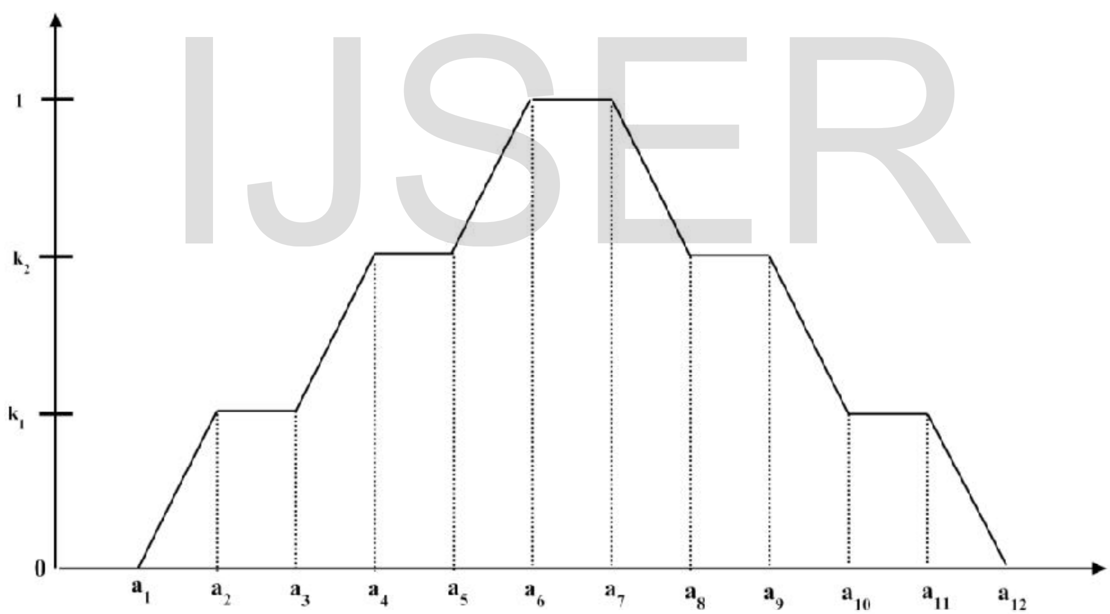

Fig: 1

4. **Ranking of Dodecagonal Fuzzy Numbers:**

Let $\tilde{A}$ be a normal dodecagonal fuzzy number. The value $M_0^{Dod}(\tilde{A})$, called the measure of $\tilde{A}$ is calculated as follows:

$$M_0^{Dod}(\tilde{A}) = \frac{1}{2}\int_1^{k_1}(f_1(r) + f_2(r))dr + = \frac{1}{2}\int_{k_1}^{k_2}(g_1(s) + g_2(s))dr + = \frac{1}{2}\int_{k_2}^{1}(h_1(t) + h_2(t))dr$$

$M_0^{Dod}(\tilde{A})=\frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)\}$ Where $0 < k_1 < k_2 < 1$

5. Mathematical Formulation of Fuzzy Game problem:

Consider a two person zero sum fuzzy game in which all the entries in the payoffs matrix are dodecagonal fuzzy numbers. Let player A has m strategies and player B has n strategies. Here it is assumed that each player has to choose from amongst the pure strategies. Player A is always assumed to be gainer and player B is always looser. The payoff matrix $m \times n$ is

$$A = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \dots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} & \dots & \tilde{a}_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3} & \dots & \tilde{a}_{mn} \end{bmatrix}$$

6. NUMERICAL EXAMPLES:

1. Consider the following fuzzy game problem with payoff as dodecagonal fuzzy numbers

$$\begin{array}{c} \\ A \end{array} \overset{B}{\begin{bmatrix} (-3,-2,-1,0,1,2,3,4,5,6,7,8) & (2,3,4,5,6,7,8,9,10,11,12,13) \\ (-5,-4,-3,-2,-1,0,1,2,3,4,5,6) & (0,1,2,3,4,5,6,7,8,9,10,11) \\ (8,9,11,12,14,15,16,17,18,21,22,23) & (2,3,5,6,8,9,10,11,12,15,16,17) \end{bmatrix}}$$

Solution: By definition of dodecagonal fuzzy number $\tilde{A}$ is calculated as $M_0^{Dod}(\tilde{A})=\frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)\}$ Where $0 < k_1 < k_2 < 1$

This problem is done by taking the values of k_1= 0.4 and k_2= 0.8 , we obtain the values of $M_0^{Dod}(\widetilde{a_{ij}})$.

$\tilde{a}_{11}$ = (−3, −2, −1,0,1,2,3,4,5,6,7,8)	$M_0^{Dod}(\tilde{a}_{11})$ = 2.5
$\tilde{a}_{12}$ = (2,3,4,5,6,7,8,9,10,11,12,13)	$M_0^{Dod}(\tilde{a}_{12})$ = 7.5
$\tilde{a}_{21}$ = (−5, −4, −3, −2, −1,0,1,2,3,4,5,6)	$M_0^{Dod}(\tilde{a}_{21})$ = 0.5
$\tilde{a}_{22}$ = (0,1,2,3,4,5,6,7,8,9,10,11)	$M_0^{Dod}(\tilde{a}_{22})$ = 5.5
$\tilde{a}_{31}$ = (8,9,11,12,14,15,16,17,18,21,22,23)	$M_0^{Dod}(\tilde{a}_{31})$ = 15.5
$\tilde{a}_{32}$ = (2,3,5,6,8,9,10,11,12,15,16,17)	$M_0^{Dod}(\tilde{a}_{32})$ = 9.5

The given fuzzy game problem is reduced in the following payoff matrix

$$\begin{array}{c} \\ A \end{array} \overset{B}{\begin{bmatrix} 2.5 & 7.5 \\ 0.5 & 5.5 \\ 15.5 & 9.5 \end{bmatrix}}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 0.5

Minimum of 3rd row = 9.5

Maximum of 1st column = 15.5

Maximum of 2nd column = 9.5

Max(min) = 9.5 and Min(max) =9.5

It has saddle point. Value of Game = 9.5

2. Consider the following fuzzy game problem :

$$A \overset{B}{\begin{bmatrix} (-3,-2,-1,0,1,2,3,4,5,6,7,8) & (6,7,8,9,10,11,12,13,14,15,16,17) & (2,3,4,5,6,7,8,9,10,11,12,13) \\ (-5,-4,-3,-2,-1,0,1,2,3,4,5,6) & (2,3,5,6,8,9,10,11,12,15,16,17) & (-5,-4,-3,-1,0,1,2,4,5,6,7,9) \\ (0,1,2,4,5,6,7,8,9,11,12,13) & (1,2,3,6,7,8,9,10,12,13,15,16) & (-2,-1,0,1,2,3,4,5,6,7,8,9,) \end{bmatrix}}$$

By definition of dodecagonal fuzzy number $\tilde{A}$ is calculated as $M_0^{Dod}(\tilde{A})=\frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)\}$Where $0 < k_1 < k_2 < 1$

Step 1. We obtain the values of M($\tilde{a}_{ij}$) of the given fuzzy game problem and convert the fuzzy game into crisp value problem which is given in the following table:

$\tilde{a}_{11} = (-3,-2,-1,0,1,2,3,4,5,6,7,8)$	$M_0^{Dod}(\tilde{a}_{11}) =$ 2.5
$\tilde{a}_{12} = (6,7,8,9,10,11,12,13,14,15,16,17)$	$M_0^{Dod}(\tilde{a}_{12}) =$ 11.5
$\tilde{a}_{13} = (2,3,4,5,6,7,8,9,10,11,12,13)$	$M_0^{Dod}(\tilde{a}_{13}) =$ 7.5
$\tilde{a}_{21} = (-5,-4,-3,-2,-1,0,1,2,3,4,5,6)$	$M_0^{Dod}(\tilde{a}_{21}) =$ 0.5
$\tilde{a}_{22} = (2,3,5,6,8,9,10,11,12,15,16,17)$	$M_0^{Dod}(\tilde{a}_{22}) =$ 9.5
$\tilde{a}_{23} = (-5,-4,-3,-1,0,1,2,4,5,6,7,9)$	$M_0^{Dod}(\tilde{a}_{32}) =$ 1.75
$\tilde{a}_{31} = (0,1,2,4,5,6,7,8,9,11,12,13)$	$M_0^{Dod}(\tilde{a}_{31}) =$ 6.5
$\tilde{a}_{32} = (1,2,3,6,7,8,9,10,12,13,15,16)$	$M_0^{Dod}(\tilde{a}_{32}) =$ 8.5
$\tilde{a}_{33} = (-2,-1,0,1,2,3,4,5,6,7,8,9,)$	$M_0^{Dod}(\tilde{a}_{33}) =$ 3.5

Step 2. The given fuzzy game problem is reduced in the following payoff matrix

$$A \overset{B}{\begin{bmatrix} 2.5 & 11.5 & 7.5 \\ 0.5 & 9.5 & 1.75 \\ 6.5 & 8.5 & 3.5 \end{bmatrix}}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 0.5

Minimum of 3rd row = 3.5

Maximum of 1st column = 6.5

Maximum of 2nd column = 11.5

Maximum of 3rd column = 7.5

Max(min) = 3.5 and Min(max) =6.5

Here Max(min) $\neq$ Min(max)

It has no saddle point.

Step 3. To solve the reduced crisp value problem ,we apply dominance method.Clearly second row is dominated by first row as all the elements of second row are less than first row. Hence eliminating second row,we get

$$A \quad \overset{B}{\begin{bmatrix} 2.5 & 11.5 & 7.5 \\ 6.5 & 8.5 & 3.5 \end{bmatrix}}$$

Again, Second column is dominated by the first column as all the elements of second column are greater than first column. Hence eliminating second column, we get

$$A \quad \overset{B}{\begin{bmatrix} 2.5 & 7.5 \\ 6.5 & 3.5 \end{bmatrix}}$$

Now we obtain 2 × 2 paypff matrix , Since the reduced matrix donnot have any saddle point , so we apply oddment method .Thus the augmented payoff matrix is

			Row oddments
	2.5	7.5	3
	6.5	3.5	5
Column oddments	4	4	8

Since the sum of row oddments and column oddments equal to 8, the optimum strategies are:

Row player ($\frac{3}{8}, \frac{5}{8}$) and column player ($\frac{1}{2}, \frac{1}{2}$)

The value of the game is 3.

7. CONCLUSION:

In this paper a method of solving fuzzy game problem using ranking of dodecagonal fuzzy numbers has been considered. The process of ranking is used to convert the fuzzy game problem into crisp value problem and then solved by traditional method.

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